MECHANICS.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

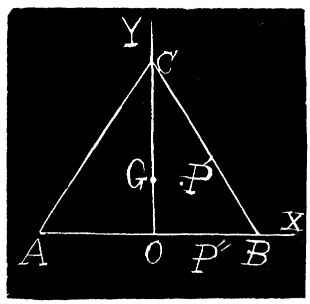
SOLUTIONS TO PROBLEMS.

5. Proposed by J. R. BALDWIN, A. M., Professor of Mathematics and Commercial Law, Davenport Business College, Davenport, Iowa.

A 200 pound ball lies on a three legged table, having the legs equally distant apart and perpendicular to the plane of the top of the table. (1) What is the weight on each leg of the table not including the top when the ball is 2 feet, 3 feet, and 4 feet distant from the three legs? (2) If the ball is 2 feet, 3 feet, and 5 feet from the legs, what must be the weight of the top to keep from tipping and the weight on each leg excluding the top and and also including the top?

Solution by ALFRED HUME, C. E., D. Sc., Professor of Mathematics in the University of Mississippi.

Let P' be the first position of ball, and A, B, and C the three legs of table.



Let these four letters, also, represent the weights at the points designated. Refer these points to the rectangular axes, OX and OY, O being the middle point of AB, and let $AB=2a$. The points A, B, C, P', will be respectively $(-a,\ 0)$,$(a,\ 0)$,$(0,\ \sqrt{3}\,a)$,$(x,\ y)$. The distances of P' from A, B, and C being 4, 2, and 3 respectively, $(x+a)^2+y^2=16$, $(x-a)^2+y^2=4$, $x^2+(y-\sqrt{3}\,a)^2=9$. From these equations, $a=2.4749$, $x=1.2122$, $y=1.5484$. (None of these values are exact, nor are any that follow).

Taking moments about OX, weight at $C=72+$.

Taking moments about OY,

$$200x=B.a-A.a=(B-A)\ a=(128-2A)a$$

and therefore, pressure at $A=15+$, and that at $B=113-$.

Let the second position of the ball be P'', the distances to A, B, and C being respectively, 3, 2, and 5. By process similar to the above the new coordinates of P are found to be, (0.5051,0.346).

If G is the center of gravity of the table and if G denotes the weight of the table then, in order that turning may not occur,

$$G\times\frac{\sqrt{3}\,a}{3}=P\times 0.346,\text{ and } G=48+.$$

Neglecting the weight of the table a force of 16 pounds must be applied downwards at C to balance P.

Then the sum of pressures at A and $B=216$, and,taking moments about OY, $(B-A)a=200x$,

$$(216-2A)a=200x,$$
$$A=88,\ B=128.$$

When weight of table is considered $A=104$, $B=144$, $C=0$.

Summing up these results,

with weight at F'................$A=15$, $B=113$, $C=72$.
" " " F' (neglecting G) $A=88$, $B=128$, $C=-16$.
" " " F'' (considering G) $A=104$, $B=144$, $C=0$.

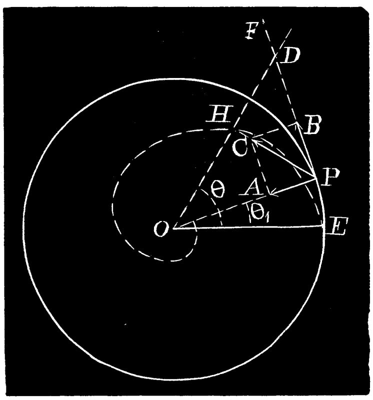
4. **Proposed by DeVOLSON WOOD, M. A., C. E., Professor of Mechanical Engineering, Stevens Institute of Technology, Hoboken, New Jersey.**

A particle starts at rest and revolves in a circle with a uniform acceleration, acquiring a velocity v in t seconds. Find the locus of the foot of the perpendicular from the centre of the circle upon the resultant acceleration.

A graphical solution by ALFRED HUME, C. E., D. Sc., Professor of Mathematics in the University of Mississippi.

CONSTRUCTION OF LOCUS.

Let the particle start from rest at E arriving at any point P in time, t, with velocity v. Let PB, PA, and PC be respectively the tangential, radial, and resultant, accelerations.



Then $PB=\frac{v}{t}$, $PA=\frac{v^2}{R}$ (numerically), and, since the acceleration in the circular path is uniform, arc $PE=\frac{1}{2}vt$.

Now, drawing OH perpendicular to PC and denoting angles HOE and POE by θ and θ_1 respectively, arc $PE=R\theta_1=\frac{1}{2}vt$, and

$$\tan(\theta-\theta_1)=\tan CPB=\frac{BC}{BP}=\frac{v^2}{R}\cdot\frac{t}{v}=\frac{vt}{R}=\frac{2R\theta_1}{R}=2\theta_1.$$

Therefore $PD=2$ arc PE, D being the intersection of PB and OH, both produced.

Hence the construction:—To find the point on required locus corresponding to P, any position of the particle, lay off on the tangent PF a distance PD equal to twice the arc PE; connect D with the center of the circle O; from P drop a perpendicular to OD meeting OD at H. H is the point required.

II. **Solution by E. C. MURPHY, C. E., University of Kansas, Lawrence, Kansas.**

Let Fig. 1 represent a circle of radius R on the circumference of which a particle is moving with a unform acceleration p, having started from rest at E. Let P be the position of the particle after time t when its velocity is V and its normal acceleration is $\frac{V^2}{R}$.